COROLLARY. *Each of the four points O, P_1, P_2, P_3, is the orthocenter of the triangle of the other three, and the set of points has all the well-known properties of an orthocentric system.*

Singularly enough, this remarkable theorem appears to be new. A rather cursory search in several of the treatises on modern elementary geometry fails to disclose it, and the author has not yet found any person to whom it was known. On the other hand, the figure is so simple (especially as it can be drawn and the theorem verified with a coin or other circular object) that it seems almost out of the question that the fact can have escaped detection. Even if geometers have overlooked it, someone must have noticed it in casually drawing circles. But if this were the case, it seems like a theorem of sufficient interest to receive some prominence in the literature, and therefore ought to be well known. It is hoped that if any reader recognizes the theorem, or knows where it has already been given, he will report the same. Of course, the converse theorem that the four circumcircles of an orthocentric system are equal, is well known.

REMARKS ON THE FOREGOING CIRCLE THEOREM.

By ARNOLD EMCH, University of Illinois.

1. The foregoing theorem proved by Mr. Johnson gains additional interest in connection with the theory of circular inversion.

Before this fact is pointed out, another proof of the theorem, equally notable on account of its extreme simplicity, will be given. Using the same notation as Mr. Johnson, and denoting the given circles through O by $\alpha_1, \alpha_2, \alpha_3$, as shown[1] in Fig. 1, we have $\measuredangle OP_3P_2 = \measuredangle OP_1P_2$, because the circles α_1 and α_3 are equal and have the common chord OP_2 subtending those angles. Likewise, $\measuredangle OP_2P_3 = \measuredangle OP_1P_3$. Consequently $\measuredangle OP_3P_2 + \measuredangle OP_2P_3 = \measuredangle OP_1P_2 + \measuredangle OP_1P_3 = \measuredangle P_2P_1P_3$, so that $\measuredangle P_2OP_3$ and $\measuredangle P_2P_1P_3$ are supplementary. But also $\measuredangle P_2AP_3$ and $\measuredangle P_2OP_3$ are supplementary (A is any point on α_1); hence $\measuredangle P_2P_1P_3 = \measuredangle P_2AP_3$. From this follows immediately that the circle α_4 through $P_1P_2P_3$ is equal to the circle α_1, and consequently to α_2 and α_3. As the sum of the six angles in the equality

$$\measuredangle OP_2P_1 + \measuredangle OP_2P_3 + \measuredangle OP_3P_2 = \measuredangle OP_3P_1 + \measuredangle OP_1P_3 + \measuredangle OP_1P_2$$

is equal to the sum of the angles in the triangle $P_1P_2P_3$, the left and right hand sides are equal to a right angle, *i. e.*, P_2P_3Q is a right triangle, and consequently P_3Q is perpendicular to P_1P_2. Likewise P_2O and P_1O prolonged are perpendiculars to P_1P_3 and P_3P_2, respectively; and O is the orthocenter of the triangle $P_1P_2P_3$. In a similar manner it can be shown, that P_1, P_2, P_3, are orthocenters of the corresponding triangles OP_2P_3, OP_3P_1, OP_1P_2.

[1] For figures in which O is without the triangle $P_1P_2P_3$, the proposition can be proved in a similar manner by angular relations.

2. Consider in Fig. 2 any triangle $P_1'P_2'P_3'$ with its circumscribed circle α_4' and the inscribed circle I with center O. Denote the sides of the triangle respectively by α_1', α_2', α_3', and invert the whole figure with respect to I as the circle of inversion. Let Q_1, Q_2, Q_3 be the points of tangency of the inscribed circle with the sides of the triangle; then α_1', α_2', α_3', supposed to be indefinitely extended,

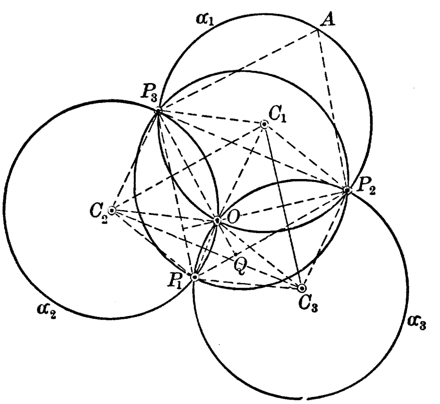

Fig.1

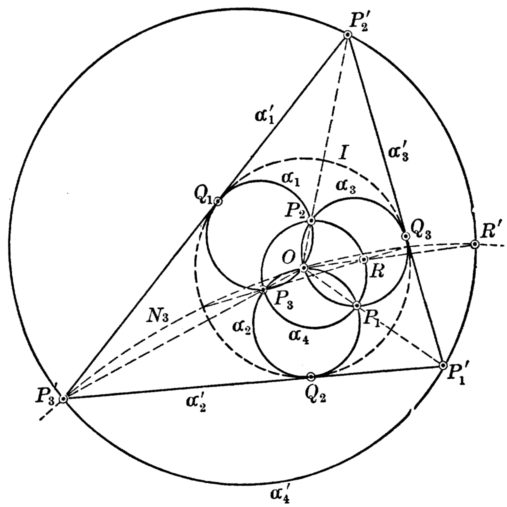

Fig.2

are inverted into three circles α_1, α_2, α_3, which pass through O and touch I at Q_1, Q_2, Q_3, respectively. They intersect in three points P_1, P_2, P_3 which in the same order are the inverse of P_1', P_2', P_3', and are evidently equal circles. The circle α_4 through P_1, P_2, P_3 is the inverse of the circumscribed circle α_4' and, according to the circle theorem, is equal to α_1, α_2, α_3. We have therefore the

THEOREM: *The indefinitely extended sides of a triangle and its circumscribed circle are inverted into four equal circles, with the inscribed* (*or escribed*) *circle of the triangle as the circle of inversion.*

Conversely, the figure of the circle theorem (Fig. 1) may always be inverted into a triangle with its in- (or escribed) and circumscribed circles.

3. Among the number of other propositions which may be derived from this theorem I shall prove only one.

Through any of the vertices, say P_3', of the triangle (Fig. 2) and the point O pass a circle N_3 cutting α_4' orthogonally, and let R' be the other point of intersection of N_3 with α_4'. The question is, what relation does the point R' have with respect to the triangle? Invert N_3 with I as the circle of inversion. The inverse is a straight line P_3R which intersects α_4 orthogonally. P_3R is therefore a diameter of α_4, and R is therefore the point of tangency of the circle having P_3 as a center and touching each of the three equal circles α_4, α_1, α_2. Inverting back, we find, that R' is the point of tangency of a circle which is tangent to the

circumscribed circle of the triangle and the two sides of the triangle that meet at P_3'. A similar result is obtained by passing circles through P_1', P_2' and O, orthogonal to α_4'. The result may be stated as the

THEOREM: *A circle which passes through a given vertex of a triangle and the center of the inscribed circle, and is orthogonal to the circumscribed circle, cuts the latter in another point which is the point of tangency of a circle that is tangent to the circumscribed circle and to the two sides of the triangle meeting at the given vertex.*

KANSAS SECTION OF THE MATHEMATICAL ASSOCIATION OF AMERICA.

The first meeting of the Kansas Section of the Mathematical Association of America was held at Lawrence, Kansas, on Saturday, March 18, 1916, in room 105, Administration Building, University of Kansas.

Following are the names of those present, together with the institutions represented:

University of Kansas: J. N. VAN DER VRIES, C. H. ASHTON, U. G. MITCHELL, H. E. JORDAN, J. J. WHEELER, S. LEFSCHETZ, A. W. LARSEN, K. L. HOLZINGER, L. L. STEIMLEY, C. A. NELSON, J. M. JACOBS, P. W. HARNLEY.

Kansas State Agricultural College: B. L. REMICK, A. E. WHITE, W. T. STRATTON, H. E. PORTER.

Kansas State Normal: THEODORE LINDQUIST.

Fairmount College: A. J. HOARE.

College of Emporia: T. E. MERGENDAHL.

Washburn College: W. A. HARSHBARGER, MARY NEWSON.

Baker University: W. H. GARRETT.

McPherson College: A. B. FRIZELL.

Bethel College: D. H. RICHERT.

Friends University: O. W. DUEKER.

Campbell College: T. L. BOUSE.

Kansas City, Kan., High School: ELIZABETH G. FLAGG, EMMA HYDE, and LUCY DOUGHERTY.

This was the second meeting of an association formed in 1915 for the improvement of the teaching of collegiate mathematics in the state of Kansas.

The Kansas Association has now become a section of the national organization, The MATHEMATICAL ASSOCIATION OF AMERICA, founded at Columbus, Ohio, December 30, 1915.

The meeting was called to order by Professor A. J. Hoare, of Fairmount College.

Professor U. G. Mitchell, of the University of Kansas, gave a report on the organization meeting of the National Association at Columbus, Ohio. Professor Mitchell was the delegate of the Kansas Section, which was the first body to make application for admission as a section of the ASSOCIATION.